(ㄱ) EDUCATION

## Axioms of Excellence: Kumon and the Russian School of Mathematics

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## Pioneer's Mission

Pioneer Institute is an independent, non-partisan, privately funded research organization that seeks to improve the quality of life in Massachusetts through civic discourse and intellectually rigorous, data-driven public policy solutions based on free market principles, individual liberty and responsibility, and the ideal of effective, limited and accountable government.

## PIONEER <br> EDUCATION

This paper is a publication of Pioneer Education, which seeks to increase the education options available to parents and students, drive system-wide reform, and ensure accountability in public education. The Center's work builds on Pioneer's legacy as a recognized leader in the charter public school movement, and as a champion of greater academic rigor in Massachusetts' elementary and secondary schools. Current initiatives promote choice and competition, schoolbased management, and enhanced academic performance in public schools.


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## Intro-background

The acronym STEM stands prominently in United States education. Grouping together the brainy disciplines of science, technology, engineering and mathematics, its excellence is considered vital for national security, world class innovation and owning the future. The competition is fierce out there, say politicians and corporate leaders, and if America is to survive it needs to produce the best.

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The math scores for the 2017 exams remained unchanged from 2015. ${ }^{2}$ Department of Education and often called "the Nation's Report Card," average math scores for eighth graders and fourth graders fell in 2015 for the first time since $1990 .{ }^{1}$ The math scores for the 2017 exams remained unchanged from 2015. ${ }^{2}$

Nor are U.S. students stacking up well against students in other countries. According to the Programme for International Student Assessment (PISA), an exam offered every three years to measure reading ability and math and science literacy skills among 15 -year-olds, the U.S. ranked $31^{\text {st }}$ out of 70 countries in 2015. In math specifically the U.S. was $39^{\text {th }}$, tied with Israel and a few points behind Slovakia and Hungary. In science the U.S. was $25^{\text {th }}$, slightly behind Norway and Portugal and a point ahead of Austria. ${ }^{3}$

A third exam conducted in 2015 was slightly more encouraging. The Trends in International Mathematics and Science Study, (TIMSS) which is given to nearly 600,000 students in dozens of education systems around the world every four years, reported that the average score of U.S. eighth graders rose slightly from results in 2011. Still, the United States trailed Singapore by 66 points in eighth-grade science and by 103 points in eighth-grade math. ${ }^{4}$

Scores from the ACT math college-entrance exam have also shown troubling declines. The graduating class of 2018 posted an average score of 20.5 , continuing a slide from the 21.1 average score of six years ago and back below the 20.6 average score recorded in 1998. ${ }^{5}$ In Massachusetts, the progress that eighth graders were once making in math in the Massachusetts Comprehensive Assessment System (MCAS) has slowed in recent years. From 2015 to 2017 the number of eighth-grade students scoring in the "Below Basic" category rose from 15 percent to 19 percent. That reversed a positive trend that began in 1992, when the percentage of students scoring at that level began to decline from a high of 37 percent. ${ }^{6}$

The causes given for this flattening of math and science outcomes vary. Poor teacher training has been cited by some critics, vague teaching standards among others, inadequate school funding by still more. In 2016 the Brown Center on Education Policy at the Brookings Institution in Washington, D.C., surveyed more than 250 international students to get their views on U.S. schools. A common opinion was that they are much less challenging than those in their home countries and that American teens are more focused on success in sports compared to their peers back home. ${ }^{7}$

A 2016 report by the National Center on Education and the Economy (NCEE) argued that U.S. elementary school students struggle with math because teachers don't know it well enough compared with top-performing countries and territories such as Finland, Japan and Hong Kong. The report stated that those countries have higher selectivity standards for teachers than the U.S. It said that, while many American primary school teachers teach all subjects, in the top countries teachers specialize in math, history or science. ${ }^{8}$

But the NCEE report is wrong. Only Finland is highly selective in its choice of teachers, and the early Finnish success on PISA was broadly criticized by its own math educators for ignoring the skills needed for math-based STEM postsecondary programs. ${ }^{9}$ Between 2003 and 2015 the PISA math scores of the Finnish students have dropped sharply. ${ }^{10}$ Elementary teachers in Japan and China do not specialize in math or history as the NCEE report claims, but rather are assigned a subject by school principals and specialize over time through on-site training and mentoring. They may also benefit from a stronger content knowledge gained in their high schools. ${ }^{11}$

Schools in many U.S. communities are investing more in teacher training, but whether that is actually paying off is an open question. Schools like the John Muir Elementary School in San Francisco think it does, yet the improvement in students' scores may be attributable more to teachers' learning the expectations of the new Smarter Balanced assessment than to the growth

> In Massachusetts, the progress that eighth graders were once making in math in the Massachusetts Comprehensive Assessment System (MCAS) has slowed in recent years. in teachers' knowledge or understanding. ${ }^{12}$ In fact, two recent and methodologically excellent studies found that intensive teacher training in mathematics had no effect on students' scores. ${ }^{13}$ The Gates' Foundation's Measures of Effective Teaching (MET) project spent three years and $\$ 575$ million to conclude that, as a sympathetic pundit put it, "We don't yet know enough about how
to find individuals who will eventually become great educators or how to train people to get there ... but one of the study's most important findings is that observations-long the core of evaluation systems-are far less predictive than anyone wanted to believe." ${ }^{14}$ In other words, knowing what makes a teacher effective, and in particular putting one's trust in yet another professional development program, may not be the wisest expenditure of money.

## After School Programs on the Rise

For many parents the solution to flat scores and poor preparation has been to turn to outside math instruction, propelling the growth of private after-school mathematics programs. National numbers are hard to come by, but in the Boston area more than a dozen private companies offer enrichment programs at nearly 50 locations. Kumon Centers, the world's largest after-school math and reading organization, has more than 1,500 locations in the U.S. serving nearly 300,000 students. The Russian School of Math, based in Newton MA, has grown to 46 locations in 11 states, plus two in Toronto, Canada since its founding in 1997.

Jon Star, an educational psychologist at the Harvard University Graduate School of Education, points to several reasons why parents are enrolling their children in after-school programs. The first is that they often have an idea of what they think math should look like, independent of what the school is engaged in. They send their kids to the after-school program to get what they believe is missing. ${ }^{15}$

That parental anxiety is also driven by a "keeping-up-with-the-Joneses" attitude, says Star, even though after-school programs thrive in high socio-economic status areas, where public education is well regarded and many students attend private schools. They enroll their children in supplemental programs despite the fact they're already receiving an excellent education.

Star says that many parents feel their kids are extraordinarily interested in math or show unusual aptitude but are not receiving the individual attention they need to maintain that interest in the classroom.
"It might be that parents feel that a particular teacher is not as skilled at giving their kids the enrichment they need or they feel their kid is super interested in math and they want to give them extra experiences in math," he says. "That's a great reason for parents to look for these extra programs." ${ }^{16}$

A third reason for their increased interest could be a
feeling on the part of some parents that their child is being poorly served by their classroom instruction regardless of their child's aptitude. Perhaps the teacher is lacking or the parents have qualms with the school's curriculum. Accordingly they decide to go elsewhere.

While Star calls the parents' concerns "legitimate," he also says it is "rare that parents have a sufficiently accurate read on the quality of instruction in the class or the quality of the curriculum being used to make a judgment to take their kids and put them in extra math for those reasons." ${ }^{17}$

Programs such as Kumon, the Russian School of Mathematics (RSM) and others have their supporters. Parents can find largely positive "reviews" from other parents of local centers on the website Yelp, as if searching for a restaurant recommendation. Four-and-a-half stars to a Kumon Center in Quincy, MA, three-and-a-half stars to a Russian School of Math center in Lexington, MA, and so on.

But there are critics as well. Hilary Kreisberg, director of the Center for Math Achievement at Lesley University, says many after-school programs promote critical thinking and deep understanding of mathematics on their web sites, but she often finds the focus has been on procedure and computation when working with their students.
"We're rooted in the same philosophy," says Kreisberg of classroom instruction and after-school programs. "We want kids to be successful and we understand that there are teaching practices that benefit students. But there is some divergence in terms of today's instruction of mathematics and the need for the critical thinking versus what is actually happening." ${ }^{18}$

This paper will look at the popularity of after-school mathematics by focusing on the Kumon and Russian School of Mathematics models. It will review their methods, highlight their best practices, and show how they complement or run parallel with mathematics taught in traditional classrooms.

## Kumon Centers

In 1954, Toru Kumon, a high school math teacher in Japan, designed a series of math worksheets to help improve the test scores of his son Takeshi, a second grader. Toru's goal was to teach Takeshi how to learn independently through the worksheets and improve his calculation skills prior to reaching high school. By working every day on the problems, Takeshi was able to reach the level of differential and integral calculus when he was just a few months into the sixth grade. ${ }^{19}$

After seeing the impact of the math sheets on Takeshi, Toru Kumon invited other local children to study at his home. Their improved skills led to the opening of an office in 1958 and a steady climb of thousands of enrolled students. After growing in Japan, Kumon expanded internationally in the 1970s, opening in New York, Taiwan, Brazil and Germany. More Kumon Centers followed in Spain, Australia, Africa and
elsewhere around the world. Today, in its $60^{\text {th }}$ year of operation, 4.2 million students are studying at nearly 25,000 learning centers in 50 countries and regions. ${ }^{20}$ In the U.S. alone, Kumon has grown from

> Parents who want to give their children a head start in math before elementary school can enroll them in Kumon as young as age 3... they can stay with the program through high school or until they complete the program. more than 182,000 students and nearly 1,300 centers in 2008 to more than 279,000 students and more than 1500 centers in 2018. ${ }^{21}$

Kumon has 45 locations in Massachusetts, 19 in or along the Route 128 beltway including Lexington, Needham, Newton, Dorchester and Brookline. They are located west of Boston in Worcester, Shrewsbury and Hopkinton; to the north in Lowell, Andover and Beverly and south to Kingston, Mansfield and Norwell.

## An early start

Parents who want to give their children a head start in math before elementary school can enroll them in Kumon as young as age 3. From that age they can stay with the program through high school or until they complete the program. Completing the program means a child has reached high school level math and can handle differential equations. How quickly they reach that level depends on each individual. Students progress as far as they can, as quickly as they can.
"We aim to get new students to grade level within 12 months of enrollment and then we aim to get them at least three years beyond grade level so we can get them to high school level math as quickly as possible," says Mary Mokris, education specialist for Kumon North America. ${ }^{22}$

Kumon touts its program as offering its greatest value for students who stick with it over time. "Parents who view Kumon as a valuable, ongoing extracurricular activity-such as guitar lessons or sports, for example-often watch their child build an academic and confidence edge that helps them achieve more, year after year in school." ${ }^{23}$

Kumon officials say they do not keep data on how their students perform on national tests such as the SAT or the before-and-after effect of their methods on student grade point averages. Mokris says the Kumon program "is individualized for each student so our results are different for each student." Student growth is assessed starting from the placement test throughout enrollment.

## Kumon describes its approach in four parts:

Individualized instruction. As a curriculum designed
to supplement school work, Kumon provides individualized instruction. New students take a placement test that measures time and accuracy on solving problems in their grade level. Instructors use the results to select the appropriate beginning math worksheets.

Self-learning. The worksheets include directions and exercises, and students work independently on them to develop self-confidence. Each student has daily assignments that take about 30 minutes to complete. Two days each week the student completes assignments at the Kumon Center. Other days the work is done at home.

Small-step worksheets. Each worksheet Kumon students are given is one-step higher in difficulty than the one before. Students attempt to build a foundation of knowledge by proving they understand each level of math before advancing. The international version of the Kumon Math Program consists of 20 levels, including preschool (from Level 6A to Level O) and five elective levels, involving a total of more than 4,400 double-sided worksheets. ${ }^{24}$

Kumon instructors. Although students work independently and are encouraged to solve problems on their own, when they are stuck on a worksheet problem Kumon instructors provide hints based upon what the student already knows.

Building self-confidence is a high priority at Kumon. When students begin the program, they're given a placement test that measures time and accuracy on solving problems in their grade level. The results are used to establish a starting point where they can begin with little difficulty. A third-grade student entering Kumon, for example, would be given a test that begins with addition and subtraction, has some multiplication and a smaller amount of division. The starting point is usually below their level of mastery so they can develop their concentration, work skills and belief in their abilities. Students progress through the math sheets at their own pace.
"(Kumon) provides individual goals based on kid-specific needs," says Jennifer Gallagher, a mother of two boys who attend the Kumon center in Hanover, MA. "Whatever they're not getting met (in school), they can fill in the gaps. I feel like school can be positive or negative and when you go there it's very positive. ${ }^{25}$
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## Math vs Applied Math

Mokris says the Kumon program is focused on "mathematics itself," as opposed to the applied mathematics found in word problems. It starts with counting and continues through differential equations and beyond. The sequential nature of the program allows young children to progress from building numeracy skills with counting to the arithmetic basics of addition, subtraction, multiplication and division, into fractions, and progressing to advanced calculus. ${ }^{26}$

Harvard's Jon Star has worked with Kumon students through his classroom research in Boston's public schools. He says that in the mathematics debate of learning concepts vs. skills, Kumon comes down on the skills side by "routinizing" important math abilities. Those proficiencies could include recall of multiplication facts, for example, or the ability to fluently do certain kinds of computations.

Parents of students attending Kumon agree that it focuses on climbing the mathematics ladder. Caitlin McCarthy, a middle school math teacher, has two daughters enrolled in the Kumon center in Norwell, MA. One is in the sixth grade and the other in the fourth grade. Both have been attending Kumon since the first grade. She sees Kumon giving them the "basic fluency they need." But Kumon is not about problem solving or critical thinking, she says.
"When my daughter in the sixth grade is doing math, she can do all the algorithms and she can factor with her eyes closed," she says. "But in the classroom she's being asked to model it and to learn more about the whys and apply it to problem solving and collaborate with group members and do some stuff on line.
"She's working on the quadratic formula," she adds. "She knows how to factor things, she knows how to apply the square root, she can rationalize denominators of square roots. She's doing all that stuff, but it's not problem solving. She's not doing it in the context of a real world problem. ${ }^{27}$

Yet in that situation she still sees Kumon as part of a larger process of educating her daughter. The Kumon method includes practicing formulas, while the classroom work teaches her how to select the correct formula to solve a problem. McCarthy says that once her daughter selects the formula "she can do the rest with her eyes closed."

Research supports McCarthy's observations. The brain can manipulate only a handful of new concepts at a time in its working (short-term) memory, but it can manipulate essentially an unlimited number of concepts/facts stored in our longterm memory. When we are exposed to a problem that has a handful of variables and operations among them, our working memory gets quickly overloaded and we tend to forget (drop)

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bits and pieces and struggle, while if we have automaticity with basic arithmetic operations, terms like "five sets of three apples" are directly held in the brain's working memory as " 15 " rather than as three "facts" (three, five, and "multiply").

In other words, arithmetic fluency greatly expands our ability to tackle more complex problems as we are not held back by flooding our working memory with trivial details. The bad name schools give to what they disdainfully call "rote memorization" and computation is actually hurting students' ability to solve higher-order problems. In that sense Kumon usefully fills an area neglected by modern American educators, who seem to have forgotten a century-old admonition of one of the giants of mathematics and philosophy.

It is a profoundly erroneous truism that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances by extending the number of important operations which we can perform without thinking about them. - Alfred North Whitehead

Hilary Kreisberg of Lesley University argues that while it might be impressive when a young student is performing well ahead of grade level, there needs to be understanding behind the performance.
"Any kid can plug in numbers to an equation and get an answer for $X$," she says. "But do they know what they're doing? If you can't explain the math and communicate the math and come up with more than one way to solve a problem, to me you're not really a mathematical thinker." ${ }^{28}$

Mokris concurs that in classrooms the Common Core State Standards typically emphasize the application of mathematics. Kumon's focus is on "pure mathematics." But she says there is a difference between rote memorization and focusing on mathematics itself. By learning something until it becomes automatic, such as the scales to play the piano, a person can then move on to more challenging levels, such as playing a composition by the $19^{\text {th }}$ century composer Carl Czerny. Learning the notes requires more memorization, but memorizing the baseline scales enables that next step.

In the Kumon method, Mokris says the brain commits facts to memory and builds strong neural pathways making it possible for the student to answer correctly, even when faced with many different problems. As with other skills people build, mathematics is learned through mastering concepts and then moving on to more challenging ones.

Many mathematicians dismiss the argument that to show real understanding of math, one must be able to explain it in words or always come up with multiple solutions. ${ }^{29}$ Many
students, particularly in early grades, find it difficult to explain the reason for every step they take in solving a problem. This often looks akin to asking a bike rider to explain in detail why he changed gears when he did, pedaled slower or faster, or how he initially got on a bike without falling. A competent rider does his movements automatically and successfully, and arguing that he "cannot be a competent rider until he can explain it" seems wrong-headed. In fact, the only thing such expectation accomplishes is to stop him from being a competent rider, as he now spends most of the time "explaining" things that he does-and must domostly instinctively and automatically. The explaining is more the role of the teacher (or the riding instructor) rather than the student.

Similarly, always expecting multiple solutions seems unnatural and counterproductive. There are many ways-algo-rithms-to successfully perform arithmetic operations, yet the point for students is to be consistently effective and successful doing it some way, rather than regularly showing he can do it in multiple ways. The purpose, after all, is to get the result of the operation to serve as the basis for the next step, rather than show off the ability to do it multiple ways. Again, we would likely be better off leaving much of the "multiple ways" to teachers rather than regularly expecting it from students. Multiple ways should be better encouraged through posing problems that naturally require different approaches, rather than through insisting on applying multiple approaches to problems that do not naturally fit them.

Parents turn to after-school programs not only as remedial work for lagging students, but for enrichment for high-achievers. They see their children excelling and want to encourage their talent or ensure that they don't grow bored by school work. The irony is that by receiving extra math tutoring after school, students advance even further ahead of their school curriculum, raising the risk they'll become uninterested in school.
"It's an unintended consequence," says Star. "In many cases, by sending your kid to the extra math, you'll make them more bored in class. I've been in math classes where the kids in the class already know the content that they're being taught that day. They're just bored because they've learned it prior to that class."

Kreisberg says the boredom risk can be offset by the classroom instruction. "If I'm creating a fun mathematics lesson and it's based on a real-world experience, and I'm showing kids how it relates to the math, that's not boring. Projects and problem-based learning is really powerful. It comes down to the planning and preparation and the thought behind it." ${ }^{30}$

Gallagher and McCarthy both had children who showed advanced math skills, such as multiplication, at a young age.

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Both were concerned that their children were becoming disengaged and turned to Kumon to supplement their school work. In each case the parents were pleased with the advanced math their children were receiving, while making it clear they still had responsibilities in school.
"He's not more ahead in school, because he still has to do the same things at school," Gallagher says of her son in the seventh grade. "The supplemental program will challenge him to the level he's at. But we explained to him that at school, he has to do his work and if he does it well, that's fine."

Coincidently, both Gallagher and McCarthy have younger children at Kumon who have less aptitude at math than their older children. With them, they see Kumon filling gaps and helping to catch up with school work.

McCarthy, the middle school math teacher, wants to ensure that her younger daughter's foundation is solid to avoid her becoming discouraged in later grades. "By eighth grade a lot of them are frustrated with math," she says. "In the elementary school, if you don't fill in the gaps, everything snowballs and they lose their confidence and by the eighth grade they aren't motivated. I feel my kids will be more apt to be challenged."

## Instructors, not teachers

Kumon Centers in the U.S. are franchises, owned and operated by the center "instructors." Though a background in education is helpful, it is not required to own a franchise. Rather, investors must have a four-year college degree and be proficient in reading and math. The estimated capital necessary to establish and market a center is $\$ 70,000$ in liquid capital and a net worth of $\$ 150,000 .{ }^{31}$

After franchise applicants pass a background and financial qualifications check, they attend a 12-day franchisee training and instructor-development program at a regional office and at Kumon University in New Jersey. Mokris says the franchisee applicants include lawyers, scientists and engineers. The development program teaches them about the materials they'll use and how to instruct students. It also trains them in the Kumon method from enrollment to ensuring the student is placed properly to making sure the instructors are correctly observing the students in the center.

Notably, Kumon instructors are not referred to as "teachers." Mokris says the term instructor is preferred to keep the focus on self-learning. Rather than teaching a class, instructors step in only when necessary and provide the materials only as needed. Mokris says Kumon instructors have a deep knowledge of the materials and can guide students to do their best self-learning.
"The instructor's job is to encourage and guide and ensure the materials are the proper ones and the student gets the correct feedback at the correct time with praise," she says.

## Russian School of Mathematics

Despite repeated requests by email and telephone to the Russian School of Mathematics for an interview, none of the founders or managers responded. Discussion of the methods and effectiveness of the school is based upon outside interviews, online articles and videos and the school's website.

While the Kumon method involves repeating mathematical processes until students over-learn them to automaticity, the Russian School of Mathematics promotes itself as believing in just the opposite.
"Russian methodology is absolutely against memorization," Inessa Rifkin, founder of RSM, said in a video posted by AmericasBestTV. "We encourage problem solving. We work on developing critical thinking and reasoning skills. We build on a deep understanding of the material. It's basically understanding versus memorization." ${ }^{32, \text { i }}$

Rifkin founded the RSM in 1997 with Irina Khavinson, a friend, educator, and fellow Russian immigrant, after concluding that her son Ilya was not receiving the same mathematics education that she received as a student in the Soviet Union. Their goal was to translate their own experiences with specialized Russian math programs into a school that offered the same opportunity to American children. ${ }^{33}$ Two decades later about 25,000 students ${ }^{34}$ are enrolled with RSM today, in 40 locations in 11 states and Canada. ${ }^{35}$

RSM has centers in 15 Massachusetts cities and towns. Fourteen of them have a median household income of \$93,600 and up, compared to the statewide average of $\$ 67,800$, according to data from the U.S. Census Bureau's American Community Survey. Five of the towns are among the 20 Massachusetts communities with the highest median income. ${ }^{36}$

In 2017 Inessa Rifkin said in an interview with National Public Radio that one-fifth of the elementary school students

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in Newton, where she founded RSM, attend the Russian School of Mathematics. ${ }^{37}$ Annual tuition at RSM can be up to $\$ 3,000 .{ }^{38}$

According to RSM website, the $11^{\text {th }}$-grade SAT average among RSM students is 774 out of 800 . It does not say if that is for a specific year or several years. The site states that students who join RSM during elementary or middle school do not require any additional courses to prepare for the SAT I or SAT II specifically. Many RSM students take the SAT in $8^{\text {th }}$ grade. "Students who join RSM in high school take a curriculum focused on building the math fundamentals required by the SAT and SAT II and by college courses." ${ }^{39}$

Russian School students placed $11^{\text {th }}$ out of 140 international teams at the Harvard-MIT Mathematics Tournament, an event held each year in February and November, though the school does not state the year in which those results were achieved. In a review we conducted of team results on the HMMT website back to November of 2014, "RSM" showed up only in November of 2017, when it placed $40^{\text {th }} .40$

Russian School students have done very well in the annual Math Kangaroo in USA mathematical competition. The multiple-choice event is for students from the first grade through grade 12. The RSM website states that "for the past three years" more than 75 percent of the Massachusetts Math Kangaroo Olympiad winners were RSM students. ${ }^{41}$ However, it does not state which years. A review of the Massachusetts results for 2017 shows that RSM students placed first, second or third-and in some cases all three-in all but the third grade. ${ }^{42}$

## Benefits of starting early

Russian School of Mathematics students attend a classroom once per week for varying lengths of time, depending on grade: 90 minutes for kindergarten through third grade; two hours for grades four through six; and two-and-a-half hours for grades seven and above. Algebra and geometry are on separate tracks starting in the sixth grade, though students may enroll in both.

School officials and teachers believe the development of reasoning skills at an early age can have a powerful impact on student development. It's an approach they say has been the "central tenet" of Russian and European education for centuries. ${ }^{43}$

Russians are not alone in recognizing the benefits of starting early. Just as many U.S. parents sign their children up for elite athletic training programs, those who can afford it are
doing the same with accelerated learning. Just as soccer camp can help an eight-year-old eventually make the high school varsity, early math training could make success in higher grades more likely as well.
"If you wait until high school to attempt to produce accelerated math learners," says Po-Shen Loh, head coach of the 2015 U.S. International Mathematical Olympiad, "the latecomers will find themselves missing too much foundational thinking and will struggle, with only four short years before college, to catch up." ${ }^{44}$

The RSM founders believe math is viewed negatively in American culture. It's considered hard to understand and tedious, they contend. Consequently, as students get older, math becomes more challenging and less fun. But the Russian approach tries to counteract that view. It treats math as "something to enjoy and marvel at, to play with, and explore." ${ }^{45}$
"We focus on taking serious math instruction and making it fun, especially in kindergarten, first and second grade," says Ilya Rifkin, chief operating officer at RSM. "We're playing with things like algebra and variables, but doing it in a way where kids aren't thinking 'I'm playing with algebra.' By the time you realize you've learned some serious math, many years have passed and you're comfortable with it." ${ }^{46}$

Slava Gerovitch, a math historian at the Massachusetts Institute of Technology, who received a master of science and a Ph.D. at universities in Moscow, says there is a distinct difference between a Russian math education and what is taught in the U.S. He points to intense teacher training and a common text book as Russian strengths.
"It's not that the Russians particularly have a gene for math or anything like that," he says. "But I think there are some systemic features of the Soviet school system that helped kids learn math easier and better. ${ }^{47}$
"The way Russians teach is that they make sure that every student, when they perform a mathematical operation, they understand why it is performed this way, not just learn how to do it," Gerovitch adds. ${ }^{48}$

Harvard's Star agrees that there is a recognition of the Russians' having a different culture than the U.S., and that somehow suggests to American parents that the way math is taught in Russia is better than the way it is taught in the U.S. He says RSM benefits from the public perception of Russian mathematicians as brilliant and "sort of a little crazy." He says some of the teachers in RSM are "very sharp and very good teachers," but the school is taking full advantage of that image of Russian mathematicians.
"The Russian School of Math has developed a cult following in the Boston area, I think, because they have these Russian emigre teachers who, when you meet them, you're blown away by them," says Star. "They're smart and engaging and your kid is really pushed to think about questions."

But he says it's the quality of the teachers, rather than the uniqueness of the teaching method that makes a difference.
"Some of their teachers are probably excellent teachers and that might be where they're getting their reputation from," he says. "But if you put those same teachers in the schools, they'd have the same positive effect. There's nothing special about the fact that they are doing the Russian school of math, it's really just that they are excellent teachers." ${ }^{49}$

Yet Star's opinion that RSM is only about quality of teachers is countered by the observed reality of different curricular content and focus. RSM strongly focuses the early years on developing understanding using abstract (non-numeric) concepts. It uses representations such as a balance to indicate equality, and stacks a variety of objects-not necessarily num-bers-on both sides to reach such equality. It then continues to develop the range of permissible operations on both sides of the balance to maintain that equality. It stresses the whole-part relationship in multiple contexts to develop deep and intuitive understanding by students who, as a result, rarely confuse what needs to be added or subtracted to another value to reach the desired outcome. Letters are introduced early as place holders for indefinite numbers, and intuitive development of the concept of area and its use to stand in for multiplication starts early. A nice way to phrase it is to say that in early grades RSM focuses on teaching mathematics rather than teaching arithmetic. ${ }^{\text {ii }}$

Fluency with basic arithmetic is not neglected, but is developed slightly later and mostly indirectly, as the result of solving a variety of problems in the classroom. Yet while such mental fluency with arithmetic is rarely explicitly targeted, it is expected and students who lack it will get an extra dose of homework to bring them up to speed. RSM's constant attention to simplifying and reducing expressions in students' work further aids developing fluency with arithmetic, while assisting in doing serious mathematics without calculators.

[^1]> But perhaps the most misunderstood aspect of RSM is its extensive use of "problems" to develop both understanding and fluency with mathematics.

But perhaps the most misunderstood aspect of RSM is its extensive use of "problems" to develop both understanding and fluency with mathematics. Modern American educators swear by solving "math problems" rather than doing "rote math" (they even, perhaps typically, cast "problem-solve" into a verb), yet what they consider a mathematical "problem" consists of a long and complex prompt (stem) that meanders through multiple sentences and is often-intentionally or not-ill-defined. When such convoluted stem is finally parsed by the student, the actual mathematics needed for the solution is trivial, typically two-to-three years below grade level. In contrast, RSM "problems" typically have a succinct and clear stem that poses a challenging mathematical question, rather than having a challenging language parsing. Below are some examples of RSM problems. ${ }^{50}$
(K-1): Jane fills a bag with three types of chips. There are 3 -point, 4 -point and 7 - point chips. Jane picks 3 chips worth 15 points. Which chips did she pick?

$$
\begin{equation*}
\frac{2010-2009+2008-2007+\cdots+2-1}{2010 \cdot 45+55 \cdot 2010} \tag{4-5}
\end{equation*}
$$

Calculate
(7-8): Prove $11 x^{2}-14 x+3 \geq 0$ if $\mathrm{x} \in \mathrm{Z}$

Some older readers may find them familiar, as that was a common type of questions decades back, before they became verbose and ill-defined in modern classrooms, often testing students' language skills more than their math skills. ${ }^{51}$ Consequently, facile claims that both RSM and Common Core focus on solving "problems" is strongly misleading.

RSM extensively uses another pedagogical tool that is rarely used these days in U.S. classrooms. Individual students approach the board and publicly explain-in writing on the board - to the rest of the class their way of solving a particular problem. The class is expected to follow and critique the solution, and offer- again, in writing on the board-improvements and corrections. The advantages should be obvious. The critique is done by other students rather than by the teacher, which makes it less threatening for students. Further, students train to be clear and explicit to allow their fellows to follow. "Arm waving," such as is often done when explanations are verbally given to a teacher, doesn't work.

Older readers may, yet again, recall doing the same long ago-or in other countries-yet this effective pedagogical
practice has largely disappeared from modern American classrooms where students are split into small groups and, at best, one representative of the group verbally explains ("arm waves," really) the group's solution to the rest of the class. The rigor of formal presentation, in writing in front of the whole class, and of formal critique, has almost disappeared, perhaps because of the fear of "shaming" students who can't clearly express themselves or hold their own. Yet this seems an indispensable tool to develop student self-assurance, clear speech, and patterns of logical presentations.

Many RSM teachers work in fields that use math, such as chemistry, meteorology and engineering, and teach parttime at RSM classrooms, according to Rifkin. ${ }^{52}$ But the backbone of the instruction is the curriculum that is consistent at all the locations. The school has a curriculum faculty team made up of seven academics with master's degrees and Ph.D.s in mathematics and applied mathematics. They provide a base of support for teachers in the classroom by developing training videos, lesson plans and teachers' notes to ensure that the concepts are properly introduced. ${ }^{53}$

Each teacher's lesson plan is reviewed and revised by a mentor. Instructors watch videos of master teachers correcting students' misunderstandings of particular concepts. Teachers also attend videoconferences to critique one another's instructional technique. ${ }^{54}$

## Conclusion and Recommendations

As Lesley University's Kreisberg points out, there is a greater understanding today that math is essential in everyday life. Jobs that pay well and are highly regarded in our changing economy often require a strong math background. Consequently, many parents feel compelled to send their children to some version of an after-school program to advance their mathematics.

Not surprisingly, at a time when U.S. student test scores are lagging behind their peers in several other major countries, two of the most successful after-school programs have their origins overseas. Kumon Centers traces its roots to Japan and a father's desire 60 years ago to improve his son's education by mastering mathematical procedure. The Russian School of

Mathematics' method is based upon the math instruction in the former Soviet Union, revived in the U.S. 20 years ago by a Russian mother and immigrant who believed her son's math instruction was lacking in their adopted country.

The two schools vary in the procedural-fluency-vs.-understanding debate. Kumon's focus on steady progress in arithmetic fluency and procedural skills contrasts with RSM's early focus on abstraction and deeper understanding. But both believe in an early start for students to develop their comfort and fluency with numbers and mental arithmetic operations, albeit in different ways, a foundation that has been neglected in many modern American classrooms.

But as Kumon, the Russian School of Mathematics and others become more popular, many public-school educators are concerned about their impact on the achievement gap and how they can work more effectively with public schools.

It seems obvious that the quality of classroom teaching students receive has a major impact on how well they learn. The U.S. Department of Education's Office for Civil Rights has identified teachers who are not fully prepared to teach math as a major factor in the achievement gap and poor student performance. Yet as we have already mentioned, while most of our elementary teachers have very high formal qualifications when compared to nations with high achieving education systems, their knowledge and understanding of elementary mathematics is seriously deficient. This points to weak teacher preparation in our teacher colleges. For example, a recent evaluation of elementary mathematics training found only one percent of traditional graduate teaching programs earned an A for adequately covering critical math content. ${ }^{55}$

In the U.S., the ratio of high-performing math students from upper income families to those from poor households is 8 to 1 . It is 3 to 1 in South Korea and 3.7 to 1 in Canada. The gains made by American students in math have mostly been limited to children of the highly educated, and largely exclude the children of the poor. By the end of high school, the percentage of low-income advanced-math learners is extremely low. ${ }^{56}$
"We know that math ability is universal and interest in math is spread pretty much equally through the population," says Daniel Zaharopol, the founder and executive director of Bridge to Enter Advanced Mathematics, a New York Citybased nonprofit organization. "We see there are almost no low-income, high-performing math students. So we know that there are many, many students who have the potential for high achievement in math but who have not had opportunity to develop their math minds, simply because they were born to the wrong parents or in the wrong zip code." ${ }^{57}$

> The U.S. Department of Education's Office for Civil Rights has identified teachers who are not fully prepared to teach math as a major factor in the achievement gap and poor student performance.

Given the paucity of competent math education in our public schools, after-school math programs seem like a viable option for quickly increasing the number of mathematically competent students. Both programs successfully do it, albeit in a very different fashion and at different depth.

## Recommendation: Increase expansion into low-income communities.

In Massachusetts, Kumon's 45 centers are primarily in mid-dle- and upper-middle class communities outside of urban areas, with the exceptions of Dorchester, Lowell and Worcester. The Russian School of Math has been even more exclusive with its expansion. The City of Framingham, with a median household income of nearly $\$ 69,000$ per year, is the only place among its 15 locations where the median income is below \$93,000.

When considering expansion Kumon, uses an array of data including median household income, population growth, minimum population, number of children age 14 and under, and other information about the area such as city planning and new housing. ${ }^{58}$

Mokris says that Kumon, with more than 1,500 centers across the U.S., has not located exclusively in upper income cities and towns. She says that Kumon partnered with schools in inner cities to provide its program to low-income families when No Child Left Behind was a federal law. Kumon also joined with schools in some specific areas of the country during its early stages of expansion in the U.S. As an example of its inner-city participation, Mokris says Kumon has a partnership with a school in Newark, N.J., although there is no longer a brick-and-mortar center in Newark. ${ }^{59}$

Options for financial assistance for after-school programs should be considered in poor-performing areas, particularly by philanthropic organizations. Across-the-board improvement of public teacher training lies beyond their capacity, but providing local areas with additional support should be attractive to some of them. Clearly, this will meet with resistance from public schools and teachers' unions, yet how can they justify keeping millions of kids behind until they clean their own house?

## Recommendation: Foster active communication between school and after-school teachers.

Kreisberg envisions communication between after-school programs and school teachers. More openness would mean teachers could inform the after-school instructors about what the students worked on during the day and what they need to be successful in the next day's lesson.
"That communication would be much more powerful," says Kreisberg. "I was a fifth-grade teacher. I never had a chance to talk with any of my students' teachers from Kumon or the Russian School of Math. The same with other similar programs. There was no teacher connection there." ${ }^{60}$

Both programs say their methods supplement some aspects of the curriculum students learn in schools, but not others. Mokris says that because Kumon is in 50 countries and regions worldwide, involving millions of students, the curriculum includes elements that are common to many school curricula. When topics are covered in schools but not in the Kumon program, she says the Kumon instruction of calculation skills enables students to handle the math.

This recommendation is also likely to meet with resistance both from public and after-school teachers. Both tend to negatively view the other side. But one hopes that creating some common forum where they can leverage rather than fight each other would be helpful.

> Options for financial assistance for after-school programs should be considered in poor-performing areas, particularly by philanthropic organizations.

## Elementary School Math Problems

Here are some challenging problems to try with your elementary school student:

## Grades K-1:

1 a.
A giraffe went on a hike. The first day he walked 29 miles. The next day he walked another 38 miles. How many miles did he walk altogether?


1 On the first day the giraffe walked X miles.
1 D. On the next day he walked another Y miles.
How many miles did he walk altogether?

2 Jane fills a bag with three types of chips. There are 3-point, 4-point and 7-point chips. Jane picks 3 chips worth 15 points. Which chips did she pick?


Russian School of Mathematics

3 The Russian School of Mathematics had an apple-eating contest. Read all the clues. Then write the correct names next to the number of apples he or she ate.

Sarah ate the most apples

Natalie ate 10 less than Sarah
Taylor did not eat more than Natalie
Nina ate more than Natalie but less than Tomer

Name
Apples Eaten
A. $\qquad$ 16
B. $\qquad$ 12
C. $\qquad$ 10
D. $\qquad$ 6
E. $\qquad$ 4


## Grades 2-4:

4 Look at the following pattern:

|  |  |
| :--- | :--- |
|  |  |


a. Draw the 17 th figure in the pattern
$\square \square$
D. Draw the 34th figure in the pattern.

|  |  |
| :--- | :--- |
|  |  |

C. Draw the 403 rd figure in the pattern.

d. Explain how to find the $n$th figure in the pattern.
$5 \begin{aligned} & \text { I know two times as many jokes as my friend. We know } 60 \text { jokes } \\ & \text { altogether, but there are three jokes that we both know. How many }\end{aligned}$ jokes do I know? How many jokes do I know that my friend doesn't?

6
I thought of a number. My friend also thought of a number. One third of my number is one half of my friend's number.

## a. Give an example of what my number could be and what my friend's number could be.

## D. Can our numbers be equal?

## 7

Jane watched her brother Peter build something that looked like stairs out of blocks. He put a block, then a tower of 2 blocks next to it, then a tower of 3 blocks... If he continues the pattern, how tall will the tallest tower be if there are 200 blocks in the kit?


## Grades 4-5:

8Three cyclists started biking around a circular track at the same time. The first completes the loop every 21 minutes. The second finishes a loop every 35 minutes and the third takes 15 minutes. How many minutes after they start will they all be together again at the starting point?


9
Calculate:

$$
\frac{2010-2009+2008-2007+\cdots+2-1}{2010 \cdot 45+55 \cdot 2010}
$$

## Middle School Math Problems

Here are some challenging problems to try with your Middle School student:

## Grades 5-6:

Pinocchio drank half a cup of black coffee. He then filled the
cup back up with milk and drank one third of the mixture.
Again he filled the cup to the top with milk, drank one sixth of the mixture and filled it back to the top with milk one final time before he drank the whole cup. Did he drink more coffee or milk?

2Simplify: $\frac{23232323}{61616161}$


3 a. Marco has a bunch of 3-peso and 5-peso bills. pesos more than seven without making change.

3 . Now Marco only has two 5-peso bills, but he still has a bunch of 3-peso bills. Can he still pay any whole number of pesos greater than seven without making change?

## Grades 7-8:

4 Prove that $5^{101}+5^{99}$ is a multiple of 13

5 Solve:
a. $(x-4)(x-5)(x-6)(x-7)=1680$
b. $x^{2}-2 x+y^{2}-4 y+5=0$
C. $\frac{1}{x^{2}+2 x-3}+\frac{18}{x^{2}+2 x+2}=\frac{18}{x^{2}+2 x+1}$

6 Prove: $11 x^{2}-14 x+3 \geq 0$ if $x \in Z$


## Endnotes

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## About Pioneer

Pioneer Institute is an independent, non-partisan, privately funded research organization that seeks to improve the quality of life in Massachusetts through civic discourse and intellectually rigorous, data-driven public policy solutions based on free market principles, individual liberty and responsibility, and the ideal of effective, limited and accountable government.


[^0]:    i It is worth observing that while Rifkin's focus on abstraction and deep understanding is characteristic of RSM, her anti-memorization sentiments seem more a marketing attempt to distance RSM from the "rote memorization" insult tossed by public school educators against after-school programs than true anti-memorization feelings. After all, everything we know-understanding, reasoning, critical thinking and all that-is based on memorization. Even reading this footnote is based on rote memorization of letter shapes and letter-sound correspondences. Only fools ignore that, and Rifkin is certainly not a fool.

[^1]:    ii It is important to distinguish between RSM's focus on abstraction and understanding that expects students to demonstrate it largely through solving complex problems, and the fashionable Common Core and similar public school approaches that focus on students' rote and verbose recitations of memorized "reasons" while solving mathematically trivial problems.

